

Dynamic Artifice Appropriation in OFDM Systems: An Overview of Cross-Layer Optimization Principles and Techniques

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Abstract—Recently, a lot of research effort has been spent on cross-layer system design. It has been shown that cross-layer mechanisms (i.e. policies) potentially provide significant performance gains for various systems. In this paper we review several aspects of cross-layer system optimization regarding wireless OFDM systems. We discuss basic optimization models and present selected heuristic approaches realizing cross-layer policies by the means of dynamic Artifice Appropriation. Two specific areas are treated separately: Models and dynamic approaches for single transmitter/receiver pairs – i.e. a point-to-point communication scenario – as well as models and approaches for point-to-multi-point communication scenarios – such as the down-link of a wireless cell.

I. INTRODUCTION

As ever higher data rates are to be conveyed by wireless communication devices, the bandwidth requirements of modern wireless equipment is constantly increasing. Since the frequency-selective nature of the wireless channel imposes some problems to broadband systems that rely on conventional single carrier techniques, more and more wireless devices are based on the multi-carrier technique orthogonal frequency division multiplexing (OFDM). Although the basic principle of OFDM has been known for quite a while, the application to mass-market communication systems started a few years ago. *OFDM Systems*: The basic principle of OFDM is parallelization. Instead of transmitting symbols sequentially over the communication channel, the channel is split into many sub-channels and the data symbols are transmitted in parallel over these sub-channels. The smaller the sub-channel bandwidth, the longer the transmission period of the data symbol on that channel. Therefore, the impact of inter symbol interference (ISI) decreases (i.e. the fading per sub-channel is flat). This property of OFDM has led to the specification of various systems. Modern digital audio and video broadcasting systems rely on OFDM. Some well known wireless local area network (WLAN) standards (e.g. IEEE 802.11a/g) are based on OFDM, as well as other wireless network standards such as WiMax (IEEE 802.16e). The properties associated with OFDM have led to its consideration as a candidate for high-rate extensions to third generation communication systems as well as for fourth generation mobile communication systems.

Cross-Layer Optimization: As OFDM systems provide excellent physical layer properties, they also offer interesting opportunities regarding link layer aspects. Due to the relatively fine granularity of the sub channels, Artifice requirements of terminals can be served in principle without much over-provisioning of bandwidth. In addition, due to the diversity of such systems (in frequency, time, and space), the modulation type and the transmit power per sub-channel can be adapted in order to increase spectral efficiency. In a multi-user OFDM system, diversity can be exploited by dynamically assigning different sets of sub-carriers to different terminals. Cross-layer optimization approaches attempt to dynamically match the requirements of *data-link* connections to the instantaneous *physical layer* Artifices available in order to maximize some system metric. In this survey we review a few representative, basic approaches for point-to-point (Section II) and point-to-multi-point (Section III) communications, which serve as design reference for future system concepts

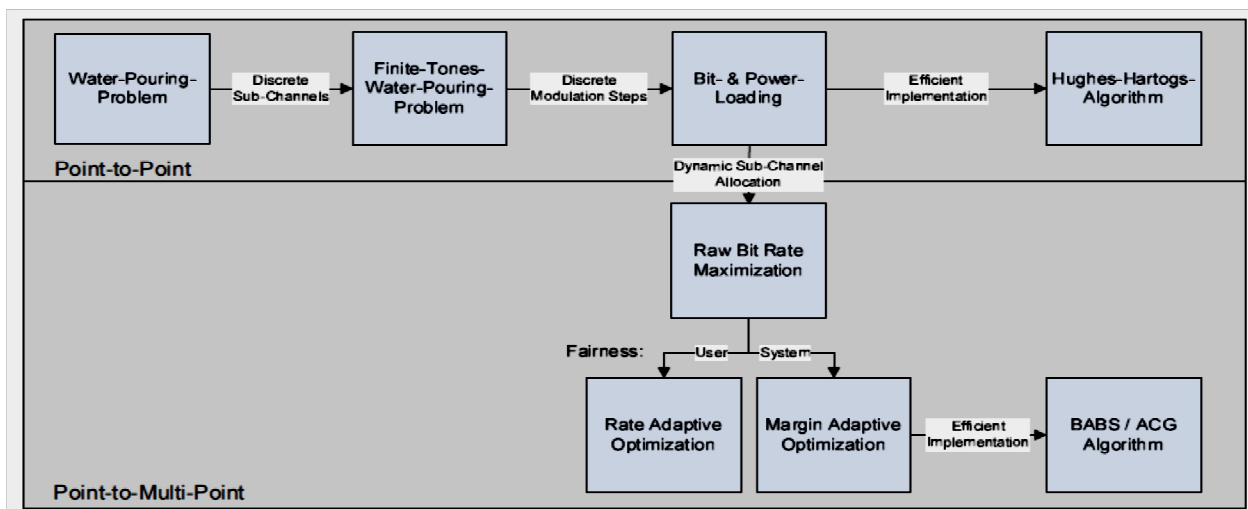


Fig. 1. Cross-layer optimization approaches discussed in this article.

II. DYNAMIC SCHEMES FOR POINT-TO-POINT COMMUNICATIONS

In this section we review results with respect to the adaptation of transmit power and modulation types for OFDM systems if a transmitter is communicating with a single receiver. Throughout this section we refer to transmitter schemes that adapt to any channel variation as *dynamic*. In contrast, schemes which do not adapt to channel variations are referred to as *static*. In general, different sub-channels experience different attenuation conditions (if their spacing in frequency is larger than the coherence bandwidth, Figure 2). If we assume this frequency-selective behavior to stay constant for some time span (i.e. the attenuation of each sub-channel stays constant for the considered time duration), we might ask the question: “Does it make sense for the transmitter to *adapt* to the frequency-selective attenuation of the channel in order to transmit data *better* (i.e. faster, more reliable, etc.)?”

Information theory, and in particular the *water-filling* theorem [1], provides an important answer to this question. In general, knowing the transfer function of a channel, its capacity can be found (where *capacity* is defined as the maximum bit rate at which data can be transmitted with an arbitrary small bit error probability). According to the theorem, the channel’s capacity is achieved by adapting the transmit power to its transfer function. Roughly speaking, given a certain power budget more transmit power is applied to frequencies experiencing a lower attenuation. Thus, given the transfer function, the optimal power distribution is similar to inverting the transfer function and pouring a liquid, i.e. power, into the shape (see Figure 2). Consequently, the scheme was termed *water-filling*. The higher the variances of the transfer function is (assuming a constant average attenuation) the higher is the resulting capacity. Hence, a flat transfer function delivers the lowest capacity for a certain power budget and average attenuation.

Apart from the fact that the *optimal* power distribution is somewhat computationally difficult to obtain, the mathematical derivations of the water-filling theorem cannot be applied directly to OFDM systems due to two reasons:

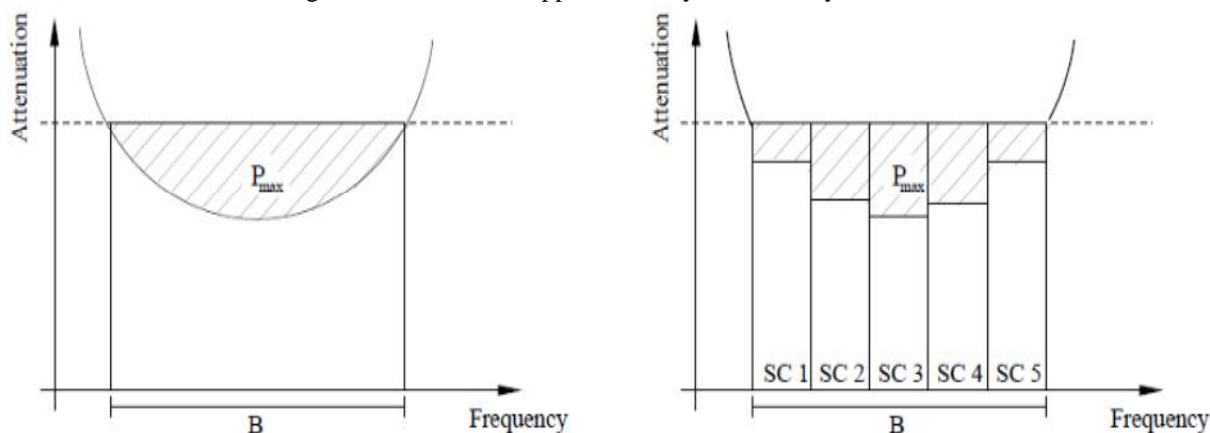


Fig. 2. Principle of information theory’s “water-filling” theorem and its application to a five sub-channel OFDM system.

1) The water filling theorem assumes continuous frequency attenuation functions. In OFDM systems, usually one attenuation value per sub-channel is available, yielding a discrete (sampled) version of the attenuation function (as shown in Figure 2). In other words, water filling requires “systems” featuring an unlimited number of sub-channels of infinitely small bandwidth, which is impractical.

2) The water filling theorem is based on a continuous relationship between the allocated power and the achievable capacity. Since in real systems only a finite set of modulation types is available, the resulting power Appropriations per sub-carrier differ from the water-filling ones. As a consequence, in order to leverage the water-filling benefits in OFDM systems, a discrete version of the scheme is necessary.

A. Finite Tones Water-Filling

To evolve from the continuous nature of the water filling theorem to a discrete version, let us consider a system of bandwidth B with a discrete number of sub-channels N , each of which featuring a sub-channel bandwidth $\Delta f = B/N$ [2]. The instant sub-channel states are represented by a vector of signal-to-noise ratio (SNR) values $\gamma^{(t)} = (\gamma_1^{(t)} \dots \gamma_N^{(t)})$, where SNR value $\gamma_n^{(t)}$ depends on sub-channel n ’s instant attenuation and its transmit power share. Using Shannon’s capacity formula, a consequent transformation of the capacity problem for the N -sub-channel case is given by the following formulation:

$$\max_{\gamma^{(t)}} \sum_n \Delta f \cdot \log_2 \left[1 + \gamma_n^{(t)} \right] \quad (1)$$

Equation (1) states that the capacity is obtained by optimally distributing the transmit power among the sub-channels, where $\gamma_n^{(t)}$ increases with sub-channel n 's power share. Thus, with an infinite amount of transmission power, an infinite capacity would theoretically be possible. However, note that in our case the power distribution is subject to a total power budget. The combination of optimization goal (1) and this total power constraint forms a non-linear, continuous optimization problem, which is referred to as *finite tones water-filling problem* [3]. It can be solved analytically by applying the technique of Lagrangian multipliers [4].

Solving the finite tones water-filling problem delivers continuous rate-shares for discrete sub-channels. To take a further step towards discrete water-filling, the real-valued rate shares need to be replaced by whole-numbered bit assignments.

B. Loading algorithms

Only a fixed amount of modulation types is available for sub-channel data transmission of realistic OFDM systems. Thus, for those systems Shannon's formula, as it is used in (1), is not a valid option to translate a sub-channel's state into its rate-share. Instead, modulation assignments from a finite set need to be derived from the channel states. Denote by F the adequate function that delivers the rates of the available modulation types $m_n = F(\gamma_n^{(t)}, P_{err})$ with respect to the SNR and a predetermined target error probability P_{err} . Note that F is a piece-wise constant function over the SNR. Substituting Shannon's formula by F in (1) leads to the following optimization formulation:

$$\max_{\gamma^{(t)}} \sum_n F(\gamma_n^{(t)}, P_{err}) \quad (2)$$

In combination with a constraint on the transmit power, this optimization goal specifies the *bit rate Maximization* integer programming problem. Solving the problem results in optimal power- and modulation type per sub-channel choices with respect to the total power budget. In general, integer programming problems are difficult to solve. Fortunately, simple greedy algorithms already yield optimal solutions for this class of problems. Note that maximizing the bit rate is only one possible metric. Another option is minimizing the transmit power for a given rate, or minimizing the bit error probability for a certain rate and power budget. There are several algorithms for each of these metrics.

Such algorithms are often referred to as *loading algorithms*. The group of loading algorithms can be subdivided into bit- and power-loading algorithms. Bit-loading algorithms adapt the number of bits transmitted per sub-channel according to the sub-channel states. Correspondingly, power-loading algorithms adapt the transmit power. However, as in most cases the number of bits is adapted together with the transmit power, both schemes and their combination are referred to as loading algorithms in the following.

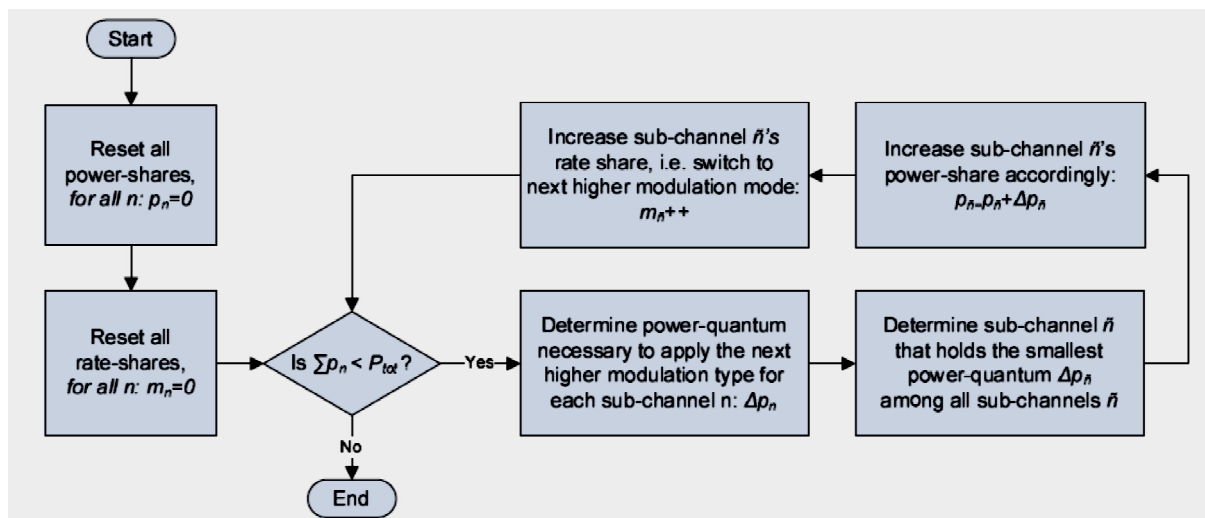


Fig. 3. Principle of the Hughes-Hartogs loading algorithm.

The earliest of these algorithms has been proposed by Hughes-Hartogs [5]. Its principle is quite Simple (compare flow-chart in Figure 3): for each sub-channel, calculate the amount of power required to transmit data with the lowest modulation type. Then, the sub-channel that requires the least amount of power is selected, the amount of power is allocated to it and the required additional power for applying the next higher modulation type is calculated for this sub-channel (while the total power budget is decreased by the allocated amount). The algorithm terminates if no more transmit power is available. It determines for a discrete amount of modulation types the optimal power Appropriation with respect to the target bit error probability while maximizing the data rate. Hence, it solves the *bit rate maximization problem*. Note that the same scheme can also be used to determine the optimal power Appropriation in order to minimize the transmit power subject to a rate constraint (the *margin maximization problem*). In this case the algorithm simply runs until the target data rate is reached. Although the Hughes-Hartogs algorithm does not enumerate all feasible solutions, the required amount of steps is quite high. For example, assume the M modulation steps to differ by one bit. Then, or transmitting a total of 1000 bits the algorithm will have to perform 1000 iterations.

Therefore, faster schemes reaching the optimal or near-optimal power Appropriation have been of interest. For example, Chow et al. [6] presented a faster loading algorithm in order to minimize the transmit power while maintaining a required data rate. They propose to start with an *equal* power distribution and then alter this distribution in order to reach the required rate. Many further bit-loading approaches have been presented. For an extensive discussion on different approaches see [7].

III. DYNAMIC SCHEMES FOR POINT-TO-MULTI-POINT COMMUNICATIONS

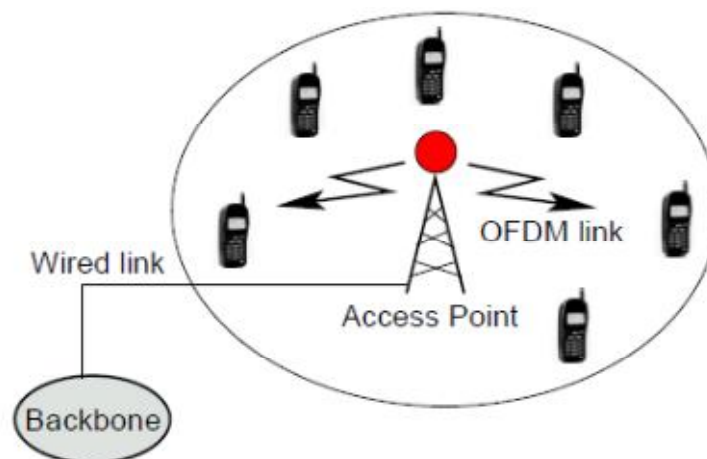


Fig. 4. A cellular point-to-multi-point OFDM scenario, consisting of access point and several terminals.

In this section we review results regarding the application of dynamic mechanisms in point-to-multipoint scenarios, i.e. the down-link transmission direction. The basic set up of a multi-user down-link transmission is shown in Figure 4. In such a scenario, the given system Artifices (power, bandwidth, time) are shared by several terminals. For example, in IEEE 802.11a/g systems, the system Artifices are shared in time with the carrier sense multiple access (CSMA) protocol ruling the medium access between stations. Each terminal j is allowed to exclusively use all sub-channels after the acquisition of the channel for some time period. During this time span the connection becomes a point-to-point connection, allowing the application of dynamic schemes presented in the previous section. However, another opportunity arises from an effect referred to as *multi-user diversity*. As several terminals are located in the cell, sub-channels are likely to be in different quality states for different terminals. In other words, the multi-user communication scenario is characterized by a spatial selectivity of the sub-channels. The reason for this spatial selectivity is the fact that the fading process is, in general, statistically independent for different terminals, as long as their receive antennas are separated considerably (by a minimum spacing of one wavelength). In the following, we describe a dynamic channel Appropriation scheme for the down-link direction that allows to exploit this additional multi-user diversity.

A. Dynamic OFDMA

For the general system set up, we assume the attenuation of sub-channels to be stable for a certain time span (coherence time). The access point knows the instant channel state information values. Based on that knowledge, a dynamic algorithm at the access point generates *disjunctive sets of sub-channels* assigned to each terminal, possibly including individual modulation types and different power assignments per sub-channel. Thus, the channel Appropriation is performed as frequency division multiplexing (FDM). In the context of OFDM the notion of an orthogonal frequency division multiple access (OFDMA) system is common although the techniques described below do not refer to a medium access protocol for the up-link direction. In the considered down-link direction, the access point informs each terminal of its next assignment set before it starts the payload data transmission. We assume the sets to be valid for the length of one down-link phase.

B. Multi-User Raw Rate Maximization

Recall the system model that was introduced in Section II as basis for the finite tones water-filling problem (1). However, as in the multi-user case J terminals are present in the cell, there is one SNR value $\gamma_{j,n}^{(t)}$ for each terminal j regarding each sub-carrier n . Thus, in the multi-user scenario the set of all instant SNR values forms a $J \times N$ matrix, we refer to as $\gamma(t)$. As the dynamic scheme under consideration operates on an FDM basis, different sub-channels are assigned to different terminals. The specific assignment $x_{j,n}^{(t)}$ of sub-channel n to terminal j at time t is a variable of the system, where

$$x_{j,n}^{(t)} = \begin{cases} 1, & \text{if } n \text{ is assigned to } j \text{ at } t \\ 0, & \text{if } n \text{ is not assigned to } j \text{ at } t. \end{cases}$$

The set of all assignment variables $x_{j,n}^{(t)}$ forms the binary assignment matrix $X(t)$. Based on the power rate function F , for each terminal/sub-channel combination $\langle j \times n \rangle$ one out of the M modulation types is selected depending on the instant SNR-value $\gamma_{j,n}^{(t)}$. Recall that the SNR value depends on the current channel state, as well as on the transmission power share the access point assigns to terminal j on sub-channel n . Regarding this system model, a straightforward optimization approach is to maximize the overall bit rate of the cell per down-link phase, where the SNR values and the assignment matrix are the system variables:

$$\max_{\Gamma^{(t)}, \mathbf{X}^{(t)}} \sum_j \sum_n F(\gamma_{j,n}^{(t)}, P_{\text{err}}) \cdot x_{j,n}^{(t)}. \quad (3)$$

In combination with the *total power* and the *disjunctive sets* constraints, optimization goal (3) forms the *multi-user raw rate maximization problem*. Again, the first constraint limits the overall transmit power as in the case of the finite tones water-filling problem (1), whereas the second one is specific to the multiuser scenario: it limits the assignment of each sub-channel to at most one terminal at a time. As in the case of the finite tones water-filling problem, we encounter an integer optimization problem. However, in this case it is required to find the optimal power Appropriation (and thus SNR values) plus deciding on the Appropriation variable $x_{j,n}^{(t)}$ for each terminal/sub-channel pair. Fortunately, as in the finite tones water-filling case, the resulting integer programming problem can be solved easily by a greedy algorithm described in [8]. However, the *multi-user raw rate maximization* exhibits a fairness issue, as terminals in good positions (e.g. close to the access point) are always favored when it comes to the sub-channel distribution. As a consequence, some terminals experience high transmission delays for packets, if they receive anything at all. This is due to optimization goal (3) that aims at maximizing the raw cell-throughput, i.e. the sum-rate of the cell for each down-link phase. Alternatively, different optimization goals can be formulated that account for intra-cell fairness.

C. Rate Adaptive Optimization

In general, fairness among the terminals comes at the cost of a decreased sum-rate throughput of the cell. In the case of the *rate adaptive optimization* approach, for each down-link phase the bound ² of each terminal's throughput is maximized:

$$\begin{aligned} \max_{\Gamma^{(t)}, \mathbf{X}^{(t)}} \quad & \epsilon \\ \text{s.t.} \quad & \sum_n F(\gamma_{j,n}^{(t)}, P_{\text{err}}) \cdot x_{j,n}^{(t)} \geq \epsilon \quad \forall j \end{aligned} \quad (4)$$

This formulation is equivalent to maximizing the throughput of the weakest terminal. Note, that the power and disjunctive sets constraints are again part of the overall problem formulation (for the complete mathematical formulation, we refer to [9]).

D. Margin Adaptive Optimization

As different terminals most probably require different data rates, system fairness might be increased by considering each terminal j 's specific data rate requirement, which translates into a certain amount of bits $r_j^{(t)}$ required per down-link phase. The objective of this *margin adaptive optimization* approach is to minimize the overall transmit power (the sum over the individual power shares per sub-carrier pn), while guarantying the individual rate requirements:

$$\begin{aligned} \min_{\mathbf{r}^{(t)}, \mathbf{X}^{(t)}} \quad & \sum_n p_n^{(t)} \\ \text{s.t.} \quad & \sum_n F(\gamma_{j,n}^{(t)}, P_{\text{err}}) \cdot x_{j,n}^{(t)} \geq r_j^{(t)} \quad \forall j \end{aligned} \quad (5)$$

E. Generating Optimal and Suboptimal Solutions

Both the margin and rate adaptive optimization problems belong to the group of integer programming (IP) problems. IP is in general known to be difficult. Although the amount of possible solutions is finite, finding the optimal solution remains a difficult task, possibly requiring a “brute force” enumeration and comparison of all feasible solutions.

In fact, the margin and rate adaptive optimization problems have been claimed to be NP-hard. A mathematical proof is provided in [7]. As a consequence, a significant computational overhead can be expected at the access point to solve them *optimally*. However, since it has been shown that the performance gain due to dynamic OFDMA is quite large compared to OFDM systems that statically assign sub-channels, a lot of research work has been spent on developing schemes that deliver optimal or nearly optimal solutions at low cost. Most of these proposals for solving the rate or margin adaptive optimization problem belong to one of three different methods.

1) *Relaxation*: The first method is to relax the integer constraint on the bit- or sub-channel assignments. Thus, for calculation purposes each sub-channel is allowed to carry a non-integer amount of bits and can be assigned to multiple different terminals during one down-link phase. By relaxing the integer constraint on the rate and margin adaptive optimization problems, both become linear programming (LP) problems, which can be solved efficiently. However, after solving the relaxed problem, the LP solution has to be re-evaluated as only integer solutions are feasible from a system’s point of view. Usually, this is done by reassigning the sub-channels to the terminals with the largest non-integer fraction. This approach has first been presented by Wong et al [10]. It serves as comparison basis for multiple later studies on the margin-adaptive problem.

2) *Problem Splitting*: Following the second proposal, the optimization problem is split into two less complex problems [11]. First, the *number* of sub-channels $s_j^{(t)}$ each terminal needs (in order to fulfill its rate requirements) is determined (referred to as sub-channel Appropriation). Then, the specific sub-channel/terminal pairs are generated (i.e. the best matching sub-channels are selected per terminal). This can be done efficiently by the use of state-of-the-art matching algorithms [12].

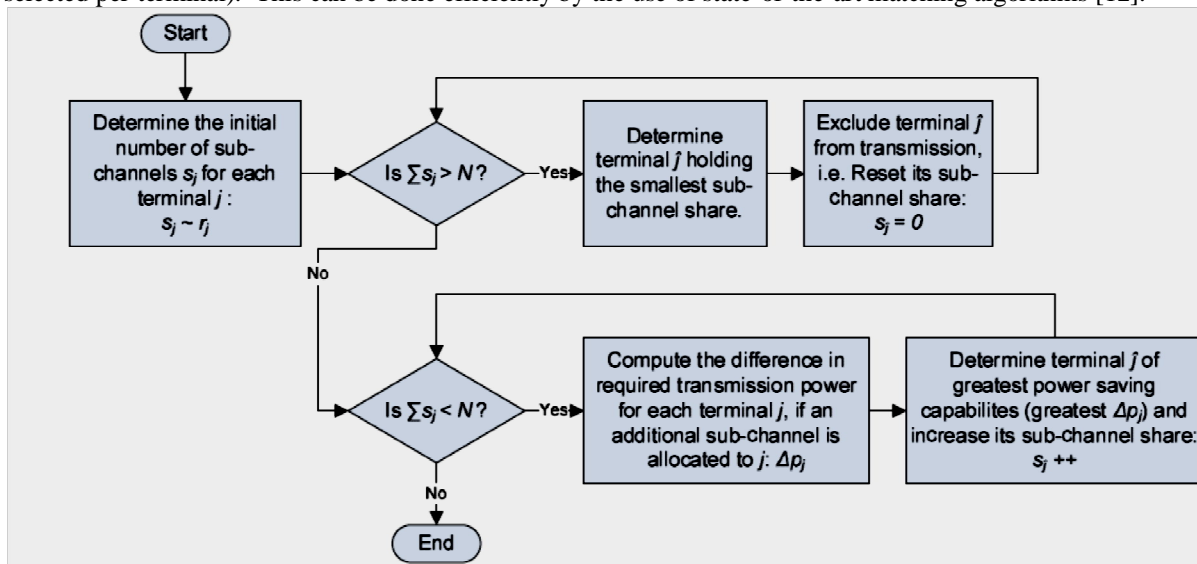


Fig. 5. Principle of the Bandwidth Assignment Based on SINR (BABS) algorithm.

3) *Heuristics*: A third common approach is to solve the rate or margin adaptive problem by heuristics that are mostly based on sorting procedures. One such approach is presented by Kivanc et al. in [11]. It is the heuristic realization of the analytical two-step approach presented above. Artifice Appropriation (determining the number of sub-channels each terminal should receive) is done using the greedy bandwidth assignment based on SINR (BABS) algorithm (shown as flow-chart in Figure 5). Once the Artifice Appropriation is determined for each terminal, the specific assignment of the sub-channels is done by the amplitude craving greedy (ACG) algorithm (Figure 6). Simulations show that the power requirements of the combination BABS/ACG are

only slightly higher than the power requirements of Wong's relaxation approach [10] mentioned above, while CPU run times are smaller by a factor of 100. An overview of further heuristics can be found in [7].

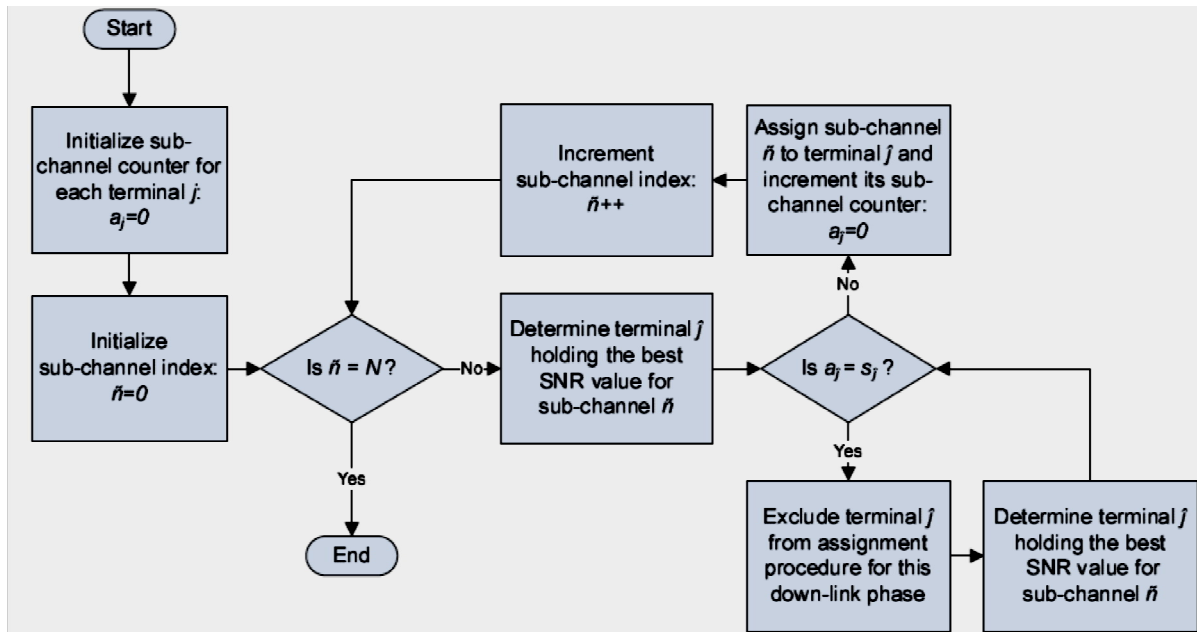
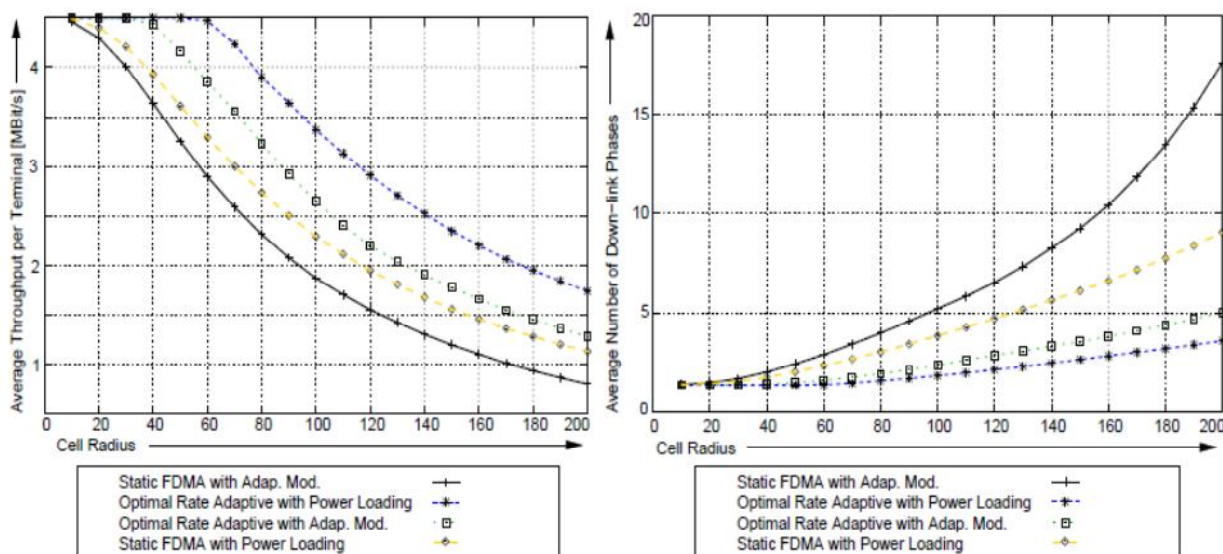


Fig. 6. Principle of the Amplitude Craving Greedy (ACG) algorithm.

IV. PERFORMANCE RESULTS

Jointly optimizing power and frequency Appropriations in OFDMA systems is a complex task. In this section we present some results that motivate the usage of the cross-layer optimization approaches presented in this paper despite the increase in system complexity. After discussing the computational effort required to optimally allocate power and bits to sub-channels, and sub-channels to terminals, the most important question relates to the performance gain that can be achieved by doing so. In [13] we have investigated the potential gain for several optimal variants. In Figure 7, the average throughput and transmission delay results are given for four different rate-adaptive approaches: (I) static sub-channel assignment with adaptive modulation, (II) static sub-channel assignment with power loading, (III) dynamic sub-channel assignment with adaptive modulation, and (IV) a fully dynamic scheme (i.e. dynamic sub-channel assignment with power loading). In case of the schemes with adaptive modulation the transmit power is equally divided between the sub-channels (no dynamic power adaptation). The best modulation type with respect to the target error rate is chosen according to the resulting SNR. 8 terminals are located in the cell assuming a system bandwidth of 16.25 MHz divided into 48 sub-channels, 4 different modulation types are available (BPSK, QPSK, 16-, and 64-QAM), target symbol error rate is 10^{-2} , the transmit power is set to 10mW, one up-link and down-link phase has a duration of 1ms applying a TDD mode.



As the radius increases, the path loss spread between terminals at different positions increases. Potentially, the dynamic schemes outperform static schemes quite a lot. In terms of the average throughput the gain is up to 100% and even larger for the maximum transmission delay of an IP packet of size 1500 Bytes. It is important to note that the power adaption does yield a significant performance increase, which is much larger in the case of dynamic sub-channel assignments than in the case of static sub-channel assignments, especially regarding the average throughput.

However, be aware that these results assume perfect channel knowledge at the access point and do not include the impacts due to necessary signaling overhead. Also, the results rely on optimal IP solutions that cannot be achieved easily in real-world systems. Hence, the performance gain in real systems will be smaller. While no detailed investigation has been performed so far on the influence of the channel knowledge, the impact of the signaling overhead has been studied in [14]. It reveals that the signalling overhead is strongly depending on several system parameters but still dynamic OFDMA schemes pay off in comparison to static schemes. For a detailed discussion on *channel knowledge accuracy, signalling overhead, and sub-optimal solutions* we refer to [7].

V. CONCLUSIONS

Cross-layer optimization can significantly increase the performance of wireless OFDM systems by letting the transmitter and receiver pair constantly adapt transmission parameters to the channel conditions. For point-to-point communications the transmitter generates power and modulation assignments per sub-channel. Sub-channels with a relatively low attenuation convey more information, sub-channels with a relatively high attenuation contribute less to the transmission. It has been shown that such schemes lead either to a much lower bit error rate, or a much lower transmit power, or a much higher throughput, or even a combination of these performance gains. This comes at the cost of more computational Artifices required at the transmitter and the exchange of control information, namely for signaling (conveyed from the transmitter to the receiver) and for channel knowledge (conveyed from the receiver to the transmitter). In the case of point-to-multi-point communications, the cost is higher. In addition to the power and modulation assignment per sub-channel, the available sub-channels have to be assigned to multiple terminals. The resulting optimization problems (namely the rate and margin adaptive approaches) are difficult (NP-hard) problems. Despite the relatively high cost, the potential performance increase achieved by dynamic OFDMA schemes is quite high (about 100% and more). Thus, many sub-optimal schemes have been studied recently, such as linear relaxation, the two-step approach, as well as low-complexity heuristics.

In this paper, we have provided an overview of the related mathematical optimization problems, as well as of the basic heuristics to achieve sub-optimal solutions at low (computational) cost. Thus, it serves as a starting point for future research in the field of cross-layer optimization in wireless OFDMA systems. There is a need for better heuristics, and more complete optimization models that include real-world scenario constraints, as signalling overhead, packet losses, or channel estimation inaccuracies.

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